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Thermodynamic modeling of creep damage in materials with different properties in tension and compression

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Abstract

A constitutive model is developed to describe the creep response of polycrystalline metals and alloys with different behavior in tension and compression. A second-order damage tensor is introduced in order to describe the creep damage under nonproportional loading in nonisothermal processes. The thermodynamic formulation of the creep equation and the damage evolution equation is used to study the creep behavior and creep damage in the initially isotropic materials. The determination of the material parameters in the proposed equations is demonstrated from a series of basic experiments outlined in this paper. The results generated from the model are compared with those obtained from experiments under uniaxial nonproportional and multiaxial nonproportional loading for both isothermal and nonisothermal processes.

Keywords: Creep; Damage; Thermodynamic modeling; Polycrystalline metals; Nonproportional loadings; Nonisothermal processes; Dislocation creep; Nucleation

1. Introduction

Since the pioneering works of Kachanov (1958) and Rabotnov (1963), quite a large number of papers on continuum creep damage mechanics have been published. However, the creep damage analysis for materials with different behavior in tension and compression has received little attention up to now even though some damage models for materials have been proposed for the case of elastic deformation (Chaboche, 1992, 1993; Gambarotta and Lagomarsino, 1993; Lubarda et al., 1994; Hansen and Schreyer, 1995; Halm and Dragon, 1996).

The creep behavior of a large class of polycrystalline materials, such as light alloys, high strength

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steels, gray cast irons, polymers, rock salt, ceramic polycrystal is the result of a number of micromechanical processes, such as the motion of defects, known as dislocations; vacancy diffusion limited by the diffusivity of the grain boundary; grain boundary sliding; vacancy diffusion limited by the rate of material transport along the void walls; migration and recrystallization, etc. (Garofalo, 1965; Tetelman and McEvily, 1967; Boyle and Spence, 1983; Rashid, 1992). The creep deformation of materials is accompanied by the growth of internal damage. There are different mechanisms of creep damage in materials (Evans, 1984; Riedel, 1987; Cocks and Leckie, 1987; Tvergaard, 1995). In the present paper, one considers creep damage that is related to creep deformation by climb-controlled dislocation motion, and to nucleation and growth of planar microcracks at the grain boundaries. That type of creep damage is the most common one for polycrystalline materials at middle and high temperatures and for engineering structures, when the design life of a component is long.

The features of creep and creep damage of polycrystalline materials may be investigated experimentally. Among these features are the following:

1. different creep properties in tension and compression;
2. damage induced anisotropy;
3. different damage development in tension and compression.

Creep behavior for many polycrystalline materials depends on the loading type (Johnson et al., 1956; Rabotnov, 1969; Tilly and Harrison, 1972; Zolochevskii, 1982; Rubanov, 1987; Chaboche, 1988; Zolochevsky, 1991; Altenbach et al., 1995). This dependency is mainly determined while comparing creep curves obtained under uniaxial tests in tension and compression at the same temperature using specimens of the same orientation. In this case, the absolute values of creep strain, chosen for one and the same absolute value of stress and for one and the same value of time, are essentially different depending on the loading type. Thus, one has two different creep curves (one in tension, and the other in compression). The maximum difference in creep behavior between tension and compression is related to the tertiary phase on the creep curves and to the corresponding creep rupture times.

On the other hand, the creep behavior is associated with a directionality of microcracks and with related damage anisotropy. If in the reference state of the material all the microcracks are randomly distributed, than under loading conditions a preferential microcrack orientation may exist. In many cases of dislocation creep of materials, the grain boundary microcracks generate and grow on planes perpendicular to the direction of the maximum positive principal stress (Johnson et al., 1956; Hayhurst, 1972; Hayhurst et al., 1980; Chen and Argon, 1981; Burger and Wilkinson, 1985). Thus, creep behavior of damaged materials is anisotropic even in the case of initial isotropy. In this regard, the creep constitutive equation must take into account the damage induced anisotropy.

Finally, the microcracks existing under creep conditions can be open (active) or closed (passive). For example, a passive damage state may occur under uniaxial compression. In other words, there is a different damage development in tension and compression, and a different orientation of microcracks under tension and compression. For the case of uniaxial tension, microcracks run perpendicular to the axis of loading (Johnson et al., 1956; Hayhurst, 1972; Burger and Wilkinson, 1985). However, under uniaxial or triaxial compression, microcracks tend to evolve and propagate in the minimum compressive stress direction (Davies and Dutton, 1966; Evans, 1984; Chan et al., 1994). A more complex picture, which is related to the closure of already existing microcracks and appearance of new ones in the perpendicular direction, emerges during the process of nonproportional loading from uniaxial tension to uniaxial compression (Davies and Dutton, 1966; Elber, 1970; Ohji and Kubo, 1988). In this case, the stress changes sign during loading, the crack closure phenomenon takes place, and the state of preexisting microcracks changes from open to closed microcracks. Consequently, the constitutive equation of creep must take into account the unilateral nature of damage and the deactivation phenomenon.

In the next sections, the proposed formulation of the theory presents further the development of the model derived by Betten et al. (1998) and takes into account simultaneously different creep properties in tension and compression, damage induced anisotropy, and different damage development in tension and compression. First, it considers the thermodynamic description of the creep behavior for damaged materials. Second, it extends the theory to nonproportional loading in nonisothermal processes. Finally, it introduces considerable improvements into the damage model derived by Betten et al. (1998). In this paper, one proposes the second-order damage tensor for describing creep damage instead of the damage vector as was earlier presented by Betten et al. (1998). In the following, one will analyze the creep behavior of initially isotropic materials within the framework of the phenomenological approach for small strains.

2. Definition of damage

Let one consider an initially isotropic polycrystalline material, in which creep damage is associated with dislocation creep, and nucleation and growth of parallel flat microcracks. The coinciding orientation of these parallel surface-like microcracks may be defined by a unit vector **n**. The microcracks are assumed to be flat, i.e. vector **n** remains constant over the microcrack.

Next, one introduces the second-order tensorial damage variable $\boldsymbol{\omega}$ with components ω_{kl} as:

$$\boldsymbol{\omega} = \omega \mathbf{n} \otimes \mathbf{n} \tag{1}$$

Here "$\otimes$" denotes the dyadic product operation, and ω is some monotonically increasing scalar parameter which can be considered as being a measure of the cumulative creep damage and can be defined from relation (1) under the condition $\mathbf{n} \cdot \mathbf{n} = 1$ as the first invariant of the damage tensor

$$\omega = \boldsymbol{\omega} \cdot \cdot \mathbf{I} = \omega_{kl} I_{kl} \tag{2}$$

where "$\cdot$" denotes the scalar product operation, and $\mathbf{I}$ is the second-order tensor with components $I_{kl} = 1$ if $k = l$ and $I_{kl} = 0$ if $k \neq l$. This damage parameter $\omega \in [0, \omega^*]$ may be useful for describing the overall creep damage of the material ($\omega \geq 0$; $\dot{\omega} \geq 0$). Here the dot above ω denotes the derivative with respect to time t. At the reference state, $t = 0$, one has $\omega = 0$, while $\omega = \omega^*$, at the instant of failure with creep.

The concept of using both damage tensor $\boldsymbol{\omega}$ and the scalar damage measure ω is similar to the concept (Krajcinovic, 1985; Lubarda et al., 1994) proposed in the damage model for brittle elastic solids. The damage tensor that describes the irreversible process of internal structure due to the growth of already existing microcracks and the appearing of new ones, is macroscopically not measurable by definition (1). On the other hand, the scalar damage parameter represents a measurable quantity. Definition for the damage parameter ω will be given later in this paper. According to Eq. (1), the damage tensor $\boldsymbol{\omega}$ is influenced by the current loading type, because the unit vector **n** changes during the deformation process. Furthermore, the damage tensor $\boldsymbol{\omega}$ changes even though the damage parameter ω does not. Obviously, the damage tensor $\boldsymbol{\omega}$ is different, when the specimen from the material under consideration is subjected to uniaxial tension, uniaxial compression or pure torsion, because different orientations of microcracks in these three tests will give rise to a different damage tensor.

One notes that the second-order tensorial damage variable was used by many authors (Murakami and Ohno, 1981; Betten, 1993; Matzenmiller and Sackman, 1994) in continuum damage mechanics. The applications of damage tensors of higher order are given by Chaboche (1982), Onat and Leckie (1988), Lubarda and Krajcinovic (1993) and Krajcinovic (1996).

3. Thermodynamic potential

One considers the description of the phenomena of elasticity, creep, damage and thermal effects for initially isotropic polycrystalline materials without creep hardening within the framework of the thermodynamics of irreversible processes (Lemaitre and Chaboche, 1990; Lemaitre, 1992). In this regard, the thermodynamic variables may be introduced as follows:

- *Observable variables*: the total infinitesimal strain tensor $\mathbf{E}$ associated with the Cauchy stress tensor $\boldsymbol{\sigma}$, and the temperature T associated with the entropy density S;
- *Internal variables*: the thermoelastic strain tensor $\mathbf{e}$ associated with the stress tensor $\boldsymbol{\sigma}$, the creep strain tensor $\boldsymbol{\varepsilon}$ associated with $-\boldsymbol{\sigma}$ due to the assumption:

$$\mathbf{E} = \boldsymbol{\varepsilon} + \mathbf{e} \tag{3}$$

and the second-order tensorial damage variable $\boldsymbol{\omega}$. Let $\mathbf{y}$ be the associated variable for $\boldsymbol{\omega}$, which will be derived later in this paper.

One assumes linear thermoelasticity without damage and uncoupling between thermoelasticity and creep. The free specific energy ψ, taken as the thermodynamic potential, is the sum of two terms: the thermoelastic part ψ_1 and the creep part ψ_2.

Then some additional comments will be made. First, to establish coupling or uncoupling between the elastic deformation and the damage evolution, one can resort to qualitative microscopic considerations (Mou and Han, 1996). From this point of view, one can say that the development of decohesion is affected by the interaction forces between atoms, and, therefore, the elastic deformation is coupled with the damage growth. Furthermore, climb-controlled dislocation motion has an effect on these interaction forces. On the other hand, from a phenomenological point of view, a creep test of very long duration at middle and high temperatures produces a significant creep strain during the application of the load, and very small thermoelastic strain. Therefore, one can neglect the damage development under elastic deformation and can consider only damage evolution under creep deformation. In other words, since the thermoelastic part ψ_1 is small in comparison with the creep part ψ_2, the effect of damage on ψ_1 will be disregarded.

Second, the microstructure of the materials under consideration cannot evolve in an unstable way with the variations in temperature. Therefore, the creep part ψ_2 in the free specific energy does not depend on the variable T. Of course, the temperature may be included in ψ_2 as a parameter.

Thus, the free specific energy has the following structure:

$$\psi = \psi_1(\mathbf{e}, T) + \psi_2(\omega) \tag{4}$$

The thermoelastic part of the free specific energy can be written as:

$$\psi_1 = \frac{1}{\rho}\left[\frac{1}{2}\left(\lambda_0 e_1^2 + 2\mu_0 e_2\right) - \left(3\lambda_0 + 2\mu_0\right)\alpha_0 \theta e_1\right] - \frac{c_0}{2T_0}\theta^2 \tag{5}$$

where $e_1 = \mathrm{tr}\,\mathbf{e} = \mathbf{e}\cdot\cdot\mathbf{I} = e_{kl}I_{kl}$, $e_2 = \mathrm{tr}\,\mathbf{e}^2 = \mathbf{e}\cdot\cdot\mathbf{e} = e_{kl}e_{kl}$, "tr" denotes the trace of a second-order tensor, ρ is the material density that can be considered to be constant, λ_0 and μ_0 are Lame's coefficients, α_0 is the coefficient of dilatation, c_0 is the specific heat at constant strain, $\theta = T - T_0$, T_0 is a reference temperature. The temperature difference θ is small with respect to T_0.

The law of thermoelasticity may be defined as:

$$\boldsymbol{\sigma} = \rho\frac{\partial\psi}{\partial\mathbf{e}} \tag{6}$$

Substituting Eqs. (4) and (5) into Eq. (6), one obtains the relation for classical linear thermoelasticity:

$$\boldsymbol{\sigma} = \lambda_0 e_1 \mathbf{I} + 2\mu_0 \mathbf{e} - \left(3\lambda_0 + 2\mu_0\right)\alpha_0 \theta \mathbf{I} \tag{7}$$

In an analogous manner, one can define the entropy density:

$$S = -\frac{\partial \psi}{\partial T} \tag{8}$$

Using Eqs. (4), (5) and (8), one obtains

$$S = \frac{1}{\rho}(3\lambda_0 + 2\mu_0)\alpha_0 e_1 + \frac{c_0}{T_0}\theta \tag{9}$$

Let one introduce the creep part ψ_2 in the free specific energy as:

$$\psi_2 = \frac{1}{\rho}\left(\omega^* - \frac{\omega \cdot \cdot \omega}{2\omega^*}\right) \tag{10}$$

Taking then into account Eq. (1), and conditions $\mathbf{n} \cdot \mathbf{n} = 1$ and $\omega \leq \omega^*$, one obtains that $\frac{\omega \cdot\cdot \omega}{\omega^*} = \frac{\omega^2}{\omega^*}$ and $\frac{\omega \cdot\cdot \omega}{\omega^*} \leq \omega^*$, and therefore, ψ_2 in Eq. (10) is a positive defined function for all types of loading. The associated variable for ω is defined as:

$$\mathbf{y} = \rho \frac{\partial \psi}{\partial \boldsymbol{\omega}} \tag{11}$$

Substituting Eqs. (4) and (10) into Eq. (11), one obtains

$$\mathbf{y} = -\frac{\boldsymbol{\omega}}{\omega^*} \tag{12}$$

4. Potentials of dissipation

Three independent dissipation potentials must be considered in order to formulate the kinetic equations describing the evolution of creep, damage and heat. This is because one has difficulty understanding why phenomena as different as creep dissipation, damage and thermal dissipation should all derive from a single potential. This approach with multiple potentials is already implied in some recent papers (Hansen and Schreyer, 1994; Chaboche, 1997).

First, one assumes the existence of a creep potential in the following form:

$$F_1 = \sigma_e^2 - \varphi_1^2(\dot{\varepsilon}_e, \omega) \equiv 0 \tag{13}$$

and

$$\dot{\boldsymbol{\varepsilon}} = \lambda \frac{\partial F_1}{\partial \boldsymbol{\sigma}} \tag{14}$$

The equivalent stress σ_e is determined by setting the equivalence of uniaxial and multiaxial stress states ($\sigma_e \geq 0$). $\dot{\varepsilon}_e$ is the equivalent creep strain rate ($\dot{\varepsilon}_e \geq 0$), and λ is a certain scalar multiplier. The equivalent stress will be defined later in this section. The equivalent creep strain rate $\dot{\varepsilon}_e$ will be considered as the scalar which is multiplied by σ_e, and yields the specific energy creep dissipation rate

$$W = \boldsymbol{\sigma} \cdot \cdot \dot{\boldsymbol{\varepsilon}} \tag{15}$$

In this regard, one has

$$W = \sigma_e \dot{\varepsilon}_e \tag{16}$$

Eq. (13) describes a certain surface in space for the stress tensor components. The first term in Eq. (13) reflects the form of this surface, and the second one describes the current position of the surface for fixed values of $\dot{\varepsilon}_e$ and ω.

The damage tensor $\boldsymbol{\omega}$ must be introduced into the creep potential (13) in order to describe damage induced anisotropy. Taking into account that the scalar damage parameter ω is already introduced into a second term of the creep potential (13), one can assume that the description of damage induced anisotropy in a first term of the creep potential is related only to the orientation $\mathbf{n}$ of the microcracks. Therefore, the equivalent stress σ_e is considered here as an isotropic invariant of the stress tensor $\boldsymbol{\sigma}$ and the dyadic product $\mathbf{n} \otimes \mathbf{n}$, i.e.

$$\sigma_e = \sigma_e(\boldsymbol{\sigma}, \mathbf{n} \otimes \mathbf{n}) \tag{17}$$

Then the integrity basis for the two symmetric second-order tensors considered here, consists of the following five invariants (Spencer, 1971):

$$\begin{aligned}
I_1 &= \operatorname{tr} \boldsymbol{\sigma} = \boldsymbol{\sigma} \cdot \cdot \mathbf{I} = \sigma_{mm}, \\
I_2 &= \operatorname{tr} \boldsymbol{\sigma}^2 = \boldsymbol{\sigma} \cdot \cdot \boldsymbol{\sigma} = \sigma_{kl}\sigma_{lk}, \\
I_3 &= \operatorname{tr} \boldsymbol{\sigma}^3 = (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}) \cdot \cdot \boldsymbol{\sigma} = \sigma_{kl}\sigma_{lm}\sigma_{mk}, \\
I_4 &= \operatorname{tr}\left[(\mathbf{n} \otimes \mathbf{n})\boldsymbol{\sigma}\right] = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} = n_k \sigma_{kl} n_l, \\
I_5 &= \operatorname{tr}\left[(\mathbf{n} \otimes \mathbf{n})\boldsymbol{\sigma}^2\right] = \mathbf{n} \cdot \boldsymbol{\sigma}^2 \cdot \mathbf{n} = n_k \sigma_{kl}\sigma_{lm} n_m.
\end{aligned} \tag{18}$$

Neglecting the influence of invariants I_3 and I_5 as second-order effects, one can introduce the linear

$$\sigma_1 = BI_4 \tag{19}$$

and quadratic

$$\sigma_2^2 = AI_1^2 + CI_2 \tag{20}$$

scalar invariant functions. Here A, B and C are material parameters dependent on the temperature. The equivalent stress can be introduced as:

$$\sigma_e = \sigma_2 + \alpha\sigma_1 \tag{21}$$

where α is a constant coefficient which takes into account the specific weight for the linear scalar function in the expression (21). It is seen that in definition (21) the equivalent stress is a homogeneous function of the stresses.

Under conditions

$$A = -\frac{1}{2}, \quad C = \frac{3}{2}, \quad \alpha = 0 \tag{22}$$

expression (21) includes as a particular case the expression for the equivalent stress

$$\sigma_{\mathrm{e}} = \sigma_{\mathrm{i}} \tag{23}$$

in the well-known Huber-von Mises-type potential for the case of isotropic materials with the same behavior in tension and compression. Here,

$$\sigma_{\mathrm{i}} = \sqrt{\frac{3}{2}\mathbf{s} \cdot \cdot \mathbf{s}} \tag{24}$$

is the stress intensity, and

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3} I_1 \mathbf{I} \tag{25}$$

is the stress deviator.

The representation of the equivalent stress in the form given by Eq. (21) is associated with the combined action of the two deformation mechanisms. The first term in Eq. (21), which reduces to the stress intensity given by Eq. (24) under the first and second conditions in Eq. (22), represents the generalization of the stress intensity in the case of the compressible materials under creep conditions and reflects the influence of the movement of dislocations on the creep behavior. The second term in Eq. (21) models the effect of microcracks with preferential orientation on the creep behavior.

One notes that an approach proposed here with the separate use of the scalar damage parameter ω and the dyadic product $\mathbf{n} \otimes \mathbf{n}$ in the creep potential (13) is contrary to an approach (Hayakawa and Murakami, 1997) in which the second-order damage tensor itself is introduced into a potential.

One may now determine the creep strain rate tensor according to the normality rule (14). By making use of Eq. (21) and differentiating, one obtains the following equation

$$\dot{\boldsymbol{\varepsilon}} = 2\lambda\sigma_{\mathrm{e}}\left(\frac{\partial\sigma_2}{\partial\boldsymbol{\sigma}} + \alpha\frac{\partial\sigma_1}{\partial\boldsymbol{\sigma}}\right) \tag{26}$$

Taking then into account the following relations

$$\frac{\partial\sigma_2}{\partial\boldsymbol{\sigma}} = \frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_2}, \quad \frac{\partial\sigma_1}{\partial\boldsymbol{\sigma}} = B\mathbf{n} \otimes \mathbf{n} \tag{27}$$

obtained from Eqs. (18)–(20), one arrives at the following equation

$$\dot{\boldsymbol{\varepsilon}} = 2\lambda\sigma_{\mathrm{e}}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_2} + \alpha B\mathbf{n} \otimes \mathbf{n}\right) \tag{28}$$

By performing the scalar product of the right- and left-hand sides of Eq. (28) with $\boldsymbol{\sigma}$, one defines the specific energy creep dissipation rate given by Eq. (15) as follows:

$$W = 2\lambda\sigma_{\mathrm{e}}^2 \tag{29}$$

Comparing Eqs. (16) and (29), one has

$$2\lambda\sigma_{\mathrm{e}} = \dot{\varepsilon}_{\mathrm{e}} \tag{30}$$

Substituting then relation (30) into Eq. (28), one obtains the constitutive equation

$$\dot{\boldsymbol{\varepsilon}} = \dot{\varepsilon}_{\mathrm{e}}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_2} + \alpha B\mathbf{n}\otimes\mathbf{n}\right) \tag{31}$$

for creep deformation related to dislocation creep and to the growth of the parallel planar microcracks. The function $\dot{\varepsilon}_{\mathrm{e}}$ in Eq. (31) may be determined from Eq. (13) as some function of the equivalent stress and of the scalar damage parameter ω through the creep curves that are obtained from basic experiments. For example, one may write:

$$\dot{\varepsilon}_{\mathrm{e}} = \frac{v(\sigma_{\mathrm{e}})}{\left(1 - \frac{\boldsymbol{\omega}}{\omega^*}\right)^m} \tag{32}$$

A power relation

$$v(\sigma_{\mathrm{e}}) = \sigma_{\mathrm{e}}^k \tag{33}$$

a hyperbolic sine rule

$$v(\sigma_{\mathrm{e}}) = \sinh\left(\frac{\sigma_{\mathrm{e}}}{d}\right) \tag{34}$$

or an exponential relation

$$v(\sigma_{\mathrm{e}}) = \exp\left(\frac{\sigma_{\mathrm{e}}}{p}\right) \tag{35}$$

are possible for the function $v(\sigma_{\mathrm{e}})$. Here m, k, d and p are temperature dependent material parameters.

One now assumes the existence of a damage potential in the following form:

$$F_2 = y_{\mathrm{e}}^2 - \varphi_2^2(\dot{\omega}, \sigma_{\mathrm{e}}) \equiv 0 \tag{36}$$

and

$$\dot{\boldsymbol{\omega}} = \mu\frac{\partial F_2}{\partial \mathbf{y}} \tag{37}$$

Here y_{e} is determined as the equivalent variable for $\mathbf{y}$ ($y_{\mathrm{e}} \geq 0$), and μ is a scalar multiplier. The equivalent variable y_{e} will be defined later in this section. Eq. (36) describes a certain damage surface in space for the components y_{kl}. The first term in Eq. (36) reflects the form of this damage surface, and the second one describes the current position of the surface for fixed values of $\dot{\omega}$ and σ_{e}.

One now introduces the variable y_{e} in the damage potential (36) as follows:

$$y_{\mathrm{e}} = 1 + \mathbf{n}\cdot\mathbf{y}\cdot\mathbf{n} \tag{38}$$

Substituting relation (12) into Eq. (38), it is easy to show that

$$y_{\mathrm{e}} = 1 - \frac{\mathbf{n}\cdot\omega\cdot\mathbf{n}}{\omega^*} \tag{39}$$

Taking then into account Eq. (1), and conditions $\mathbf{n}\cdot\mathbf{n} = 1$ and $\omega \leq \omega^*$, one has

$$y_{\mathrm{e}} = 1 - \frac{\omega}{\omega^*} \tag{40}$$

and therefore $y_{\mathrm{e}} \geq 0$. In this regard, y_{e} is always non-negative function.

One may determine the damage rate tensor according to the normality rule (37). Using Eqs. (36) and (38), and differentiating, one obtains the following equation

$$\dot{\boldsymbol{\omega}} = 2\mu y_{\mathrm{e}} \mathbf{n} \otimes \mathbf{n} \tag{41}$$

On the other hand, differentiating Eq. (2), one has

$$\dot{\omega} = \dot{\boldsymbol{\omega}} \cdot\cdot \mathbf{I} \tag{42}$$

By performing the scalar product of the right- and left-hand sides of Eq. (41) with $\mathbf{I}$ and using Eq. (42), one obtains

$$\dot{\omega} = 2\mu y_{\mathrm{e}} \tag{43}$$

Substituting relation (43) into Eq. (41), one obtains the damage evolution equation

$$\dot{\boldsymbol{\omega}} = \dot{\omega} \mathbf{n} \otimes \mathbf{n} \tag{44}$$

Taking then into account Eq. (1) and differentiating them, one may see that the damage evolution equation (44) is formulated under condition that the damage flux $\dot{\boldsymbol{\omega}}$ depends on the rate of change of the scalar damage parameter and does not depend on the variations of the unit vector $\mathbf{n}$. This condition can be assumed for materials without creep hardening under proportional and nonproportional loading.

The function $\dot{\omega}$ in Eq. (44) may be determined from Eqs. (36) and (40) as some function of the equivalent stress and of the variable y_{e} through the creep curves and stress-creep failure time data that are obtained from basic experiments. For example, one may write

$$\dot{\omega} = \frac{\chi(\sigma_{\mathrm{e}})}{\left(1 - \frac{\omega}{\omega^*}\right)^q} \tag{45}$$

Here $\chi(\sigma_{\mathrm{e}})$ is some function of the equivalent stress, and q is a temperature dependent material parameter. As it was established (Rubanov, 1987; Zolochevsky, 1991; Altenbach et al., 1995; Betten et al., 1998) for various polycrystalline materials under uniaxial and multiaxial loading, one can take in Eq. (45) $\chi(\sigma_{\mathrm{e}}) = v(\sigma_{\mathrm{e}})\sigma_{\mathrm{e}}$ and $m = q$. Using then Eqs. (15), (16) and (32), it is not difficult to obtain the following equation:

$$\dot{\omega} = \dot{\varepsilon}_{\mathrm{e}} \sigma_{\mathrm{e}} = W \tag{46}$$

Thus, the rate of change of the scalar damage parameter is equal to the specific energy creep dissipation rate.

The equality (46) may be used for identification of the scalar damage parameter $\omega = \omega(t)$ and its critical value ω^* at the instant of failure with creep in basic experiments with constant stress and constant temperature. For example, for the case of uniaxial tension with $\sigma_{11} \neq 0$, one obtains

$$\omega = \sigma_{11} \varepsilon_{11} \tag{47}$$

and

$$\omega^* = \sigma_{11} \varepsilon_{11}^* \tag{48}$$

Analogously, the relations for the case of uniaxial compression with $\sigma_{11} \neq 0$ are given by:

$$\omega = \sigma_{11}\varepsilon_{11} \tag{49}$$

and

$$\omega^* = \sigma_{11}\varepsilon_{11}^* \tag{50}$$

Considering pure torsion with $\sigma_{12} \neq 0$, one has the following relations:

$$\omega = 2\sigma_{12}\varepsilon_{12} \tag{51}$$

and

$$\omega^* = 2\sigma_{12}\varepsilon_{12}^* \tag{52}$$

Here ε_{11}^* and $2\varepsilon_{12}^*$ are critical values of the creep strain corresponding to creep rupture in basic experiments under discussion.

Finally, by considering the heat received by conduction through the boundary of the body, one assumes the existence of a thermal dissipation potential in the following form:

$$F_3 = \frac{1}{2}d_0\mathbf{g}\cdot\mathbf{g} \tag{53}$$

and

$$-\frac{\mathbf{q}}{T} = \frac{\partial F_3}{\partial \mathbf{g}} \tag{54}$$

Here $\mathbf{q}$ and $\mathbf{g}$ are the heat flux vector and its dual variable, and d_0 is a scalar parameter. In this work, $\mathbf{g}$ is given by

$$\mathbf{g} = \mathbf{grad}\ T \tag{55}$$

According to the normality rule given by Eq. (54), one obtains the following relation by using Eqs. (53) and (55)

$$\mathbf{q} = -d_0 T\ \mathbf{grad}\ T \tag{56}$$

Assuming that

$$d_0 = \frac{k_0}{T} \tag{57}$$

one may express $\mathbf{q}$ as follows:

$$\mathbf{q} = -k_0\ \mathbf{grad}\ T \tag{58}$$

where k_0 is the coefficient of thermal conductivity. It is seen that the derived Eq. (58) is the classical law of heat diffusion or Fourier's law.

One may now write the second principle of thermodynamics to ensure the validity of the model proposed here. In this regard, one uses the Clausius–Duhem inequality:

$$\Phi = \boldsymbol{\sigma}\cdot\cdot\dot{\boldsymbol{\varepsilon}} - \mathbf{y}\cdot\cdot\dot{\omega} - \mathbf{g}\cdot\frac{\mathbf{q}}{T} \geq 0 \tag{59}$$

Using Eqs. (15) and (16), one can express the first term in (59) as follows:

$$\boldsymbol{\sigma}\cdot\cdot\dot{\boldsymbol{\varepsilon}} = \sigma_{\mathrm{e}}\dot{\varepsilon}_{\mathrm{e}} \geq 0 \tag{60}$$

Considering Eqs. (1), (12) and (44), and condition $\mathbf{n}\cdot\mathbf{n}=1$, one can rewrite the second term in Eq. (59) as:

$$-\mathbf{y}\cdot\cdot\dot{\boldsymbol{\omega}} = \frac{\omega\dot{\omega}}{\omega^*} \geq 0 \tag{61}$$

Using Eqs. (55) and (58), one can finally express the third term in Eq. (59) as follows:

$$-\mathbf{g}\cdot\frac{\mathbf{q}}{T} = \frac{k_0}{T}|\mathbf{grad}\ T|^2 \geq 0 \tag{62}$$

Thus, within the proposed creep damage model the second thermodynamic principle is always valid.

5. Basic experiments

One now considers a procedure for the determination of the material parameters in the proposed constitutive equation of creep Eq. (31) and the damage evolution equation Eq. (44) together with Eqs. (32) and (46) as well as for identification of the functions $\dot{\varepsilon}_{\mathrm{e}}$ and $\dot{\omega}$. To do this, one needs the results of the basic experiments for different values of constant stress at constant temperature on standard specimens in which a homogeneous stress state is realized. In this case, one does not need to use Fourier's law given by Eq. (58), the entropy density given by Eq. (9), and the first principle of thermodynamics. The temperature field is given in the case under consideration, and consequently, one does not need to use the flux.

One assumes that for the case of uniaxial tension with $\sigma_{11}\neq 0$, one has the following relations:

$$\dot{\varepsilon}_{11} = \frac{K_+\sigma_{11}^k}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{63}$$

and

$$\dot{\omega} = \frac{K_+\sigma_{11}^{k+1}}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{64}$$

Considering uniaxial compression with $\sigma_{11}\neq 0$, one obtains

$$\dot{\varepsilon}_{11} = -\frac{K_-|\sigma_{11}|^k}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{65}$$

and

$$\dot{\omega} = \frac{K_-|\sigma_{11}|^{k+1}}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{66}$$

Analogously, the relations for the case of pure torsion with $\sigma_{12}\neq 0$ are given by:

$$2\dot{\varepsilon}_{12} = \frac{K_0\sigma_{12}^k}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{67}$$

and

$$\dot{\omega} = \frac{K_0 \sigma_{12}^{k+1}}{\left(1 - \frac{\omega}{\omega^*}\right)^m} \tag{68}$$

Here K_+, K_-, K_0, m, and k are temperature dependent material constants. It is seen that the powers m and k do not depend on the type of loading. This fact was experimentally established (Zolochevskii, 1982; Rubanov, 1987; Zolochevsky, 1991; Altenbach et al., 1995) for various polycrystalline materials under consideration. It was also established (Rubanov, 1987; Zolochevsky, 1991; Altenbach et al., 1995; Betten et al., 1998) that Eqs. (63)–(68) describe well the creep deformation of various damaged materials under uniaxial tension, uniaxial compression, and pure torsion.

On the other hand, one can use Eqs. (31), (32), (44) and (46) to write the relations describing creep and creep damage in these basic experiments. Since the unit vector **n** does not represent a measurable quantity, one will assume that microcracks in the material are always orthogonal to the direction of the maximum principal stress. One can also take the power function (33) for representation of the functions $\dot{\varepsilon}_e$ and $\dot{\omega}$ given by Eqs. (32) and (46).

One notes that different indirect methods for identification of the orientation of microcracks in the material under multiaxial loading were discussed by Chaboche (1993).

Under uniaxial tension ($\sigma_{11} > 0$) microcracks propagate perpendicular to the axis of loading. In this regard, the orientation of microcracks is represented by the vector $\mathbf{n} = [1, 0, 0]^T$. Therefore, the components of the damage tensor given by representation (1) can be expressed for the case of uniaxial tension as follows:

$$\omega_{11} = \omega, \quad \omega_{22} = \omega_{33} = \omega_{12} = \omega_{13} = \omega_{23} = 0 \tag{69}$$

Then it is not difficult to obtain from Eqs. (31)–(33), (44) and (46) the following relations:

$$\dot{\varepsilon}_{11} = \frac{\left(\sqrt{A+C} + \alpha B\right)^{k+1} \sigma_{11}^k}{\left(1 - \frac{\omega}{\omega^*}\right)^m} \tag{70}$$

and

$$\dot{\omega} = \frac{\left(\sqrt{A+C} + \alpha B\right)^{k+1} \sigma_{11}^{k+1}}{\left(1 - \frac{\omega}{\omega^*}\right)^m} \tag{71}$$

Comparing Eqs. (63) and (70) and (64) and (71), one has

$$\left(\sqrt{A+C} + \alpha B\right)^{k+1} = K_+ \tag{72}$$

In the case of uniaxial compression ($\sigma_{11} < 0$) microcracks form and propagate parallel to the axis of compressive loading. Therefore, one has $\mathbf{n} = [0, 1, 0]^T$. Taking into account the representation (1) for the damage tensor, one obtains

$$\omega_{22} = \omega, \quad \omega_{11} = \omega_{33} = \omega_{12} = \omega_{13} = \omega_{23} = 0 \tag{73}$$

Then one can rewrite Eqs. (31)–(33), (44) and (46) for this stress state as follows:

$$\dot{\varepsilon}_{11} = -\frac{\left(\sqrt{A+C}\right)^{k+1} |\sigma_{11}|^k}{\left(1 - \frac{\omega}{\omega^*}\right)^m} \tag{74}$$

and

$$\dot{\omega} = \frac{\left(\sqrt{A+C}\right)^{k+1}|\sigma_{11}|^{k+1}}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{75}$$

Equating the expressions, given in Eqs. (65) and (74) and (66) and (75), one obtains

$$\left(\sqrt{A+C}\right)^{k+1} = K_- \tag{76}$$

Under pure torsion in the 1–2 plane ($\sigma_{12} \neq 0$), microcracks run at an angle of $\pi/4$ with respect to axes 1, 2. Therefore, the orientation of microcracks is represented by the vector $\mathbf{n} = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^{\mathrm{T}}$. In this regard, the components of the damage tensor are defined as:

$$\omega_{11} = \omega_{22} = \omega_{12} = \frac{1}{2}\omega, \quad \omega_{33} = \omega_{13} = \omega_{23} = 0 \tag{77}$$

Then it is not difficult to obtain from Eqs. (31)–(33), (44) and (46) for the case under consideration the following relations:

$$2\dot{\varepsilon}_{12} = \frac{\left(\sqrt{2C}+\alpha B\right)^{k+1}\sigma_{12}^k}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{78}$$

and

$$\dot{\omega} = \frac{\left(\sqrt{2C}+\alpha B\right)^{k+1}\sigma_{12}^{k+1}}{\left(1-\frac{\omega}{\omega^*}\right)^m} \tag{79}$$

Equating Eqs. (67) and (78) and (68) and (79), yields the following expression

$$\left(\sqrt{2C}+\alpha B\right)^{k+1} = K_0 \tag{80}$$

It is not difficult to determine from (72), (76) and (80) the material parameters as follows:

$$\alpha B = K_+^{\frac{1}{k+1}} - K_-^{\frac{1}{k+1}}, \quad \sqrt{2C} = K_0^{\frac{1}{k+1}} - \alpha B, \quad A = K_-^{\frac{2}{k+1}} - C \tag{81}$$

6. Particular cases

One now considers various particular cases resulting from the constitutive relation of creep given by Eq. (31) and from the damage evolution law given by Eq. (44) together with Eqs. (32), (33) and (46). The data of the basic experiments are used together with Eq. (81).

One now assumes that from the basic experiments one obtains

$$K_- = K_+, \quad K_0 = 3^{\frac{k+1}{2}} K_+ \tag{82}$$

Substituting Eq. (82) into Eq. (81), one arrives at the following relations

$$\alpha = 0, \quad C = \frac{3}{2}K_{+}^{\frac{2}{k+1}}, \quad A = -\frac{1}{3}C \tag{83}$$

The equivalent stress given by Eq. (21) can now be rewritten on the basis of the stress intensity given by Eq. (24), i.e.

$$\sigma_{\mathrm{e}} = \sqrt{\frac{2}{3}C}\sigma_{\mathrm{i}} \tag{84}$$

while the proposed Eqs. (31) and (44) together with relations (32), (33) and (46) become

$$\dot{\boldsymbol{\varepsilon}} = \frac{\left(\sqrt{\frac{2}{3}C}\sigma_{\mathrm{i}}\right)^{k}}{\left(1 - \frac{\omega}{\omega^{*}}\right)^{m}}\sqrt{\frac{3}{2}C}\frac{s}{\sigma_{\mathrm{i}}} \tag{85}$$

and

$$\dot{\boldsymbol{\omega}} = \frac{\left(\sqrt{\frac{2}{3}C}\sigma_{\mathrm{i}}\right)^{k+1}}{\left(1 - \frac{\omega}{\omega^{*}}\right)^{m}}\mathbf{n} \otimes \mathbf{n} \tag{86}$$

The resulting Eqs. (85) and (86) correspond to the traditional equations for isotropic materials which are insensitive to the loading type. In this regard, the equalities in Eq. (82) are recommendations for using Eqs. (85) and (86) which do not account for the different creep behavior in tension–compression, the different damage development in tension–compression, and the independent creep law and independent damage law of isotropic materials under conditions of pure torsion. Non-existence of even one of the equalities in Eq. (82) shows the impossibility of using Eqs. (85) and (86).

One assumes that the following data are obtained from the basic experiments

$$K_{-} = K_{+}, \quad K_{0} \neq 3^{\frac{k+1}{2}}K_{+} \tag{87}$$

Using Eq. (81) together with Eq. (87), one arrives at the following relations

$$\alpha = 0, \quad C = \frac{1}{2}K_{0}^{\frac{2}{k+1}}, \quad A = K_{+}^{\frac{2}{k+1}} - \frac{1}{2}K_{0}^{\frac{2}{k+1}} \tag{88}$$

Then the equivalent stress given by expression (21) takes the form

$$\sigma_{\mathrm{e}} = \sigma_{2} \tag{89}$$

while the proposed Eqs. (31) and (44) together with relations (32), (33) and (46) are given as follows:

$$\dot{\boldsymbol{\varepsilon}} = \frac{\sigma_{2}^{k}}{\left(1 - \frac{\omega}{\omega^{*}}\right)^{m}}\left(\frac{AI_{1}\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_{2}}\right) \tag{90}$$

and

$$\dot{\boldsymbol{\omega}} = \frac{\sigma_{2}^{k+1}}{\left(1 - \frac{\omega}{\omega^{*}}\right)^{m}}\mathbf{n} \otimes \mathbf{n} \tag{91}$$

In order to describe the behavior of these isotropic damaging materials with the same properties in tension and compression, one needs to use creep and damage characteristics in tension and torsion.

One assumes that from the basic experiments one obtains

$$K_{-} \neq K_{+}, \quad K_0 = \left[\left(\sqrt{3} - 1\right) K_{-}^{\frac{1}{k+1}} + K_{+}^{\frac{1}{k+1}} \right]^{k+1} \tag{92}$$

Substituting Eq. (92) into Eq. (81), one obtains the following relations:

$$\alpha B = K_{+}^{\frac{1}{k+1}} - K_{-}^{\frac{1}{k+1}}, \quad C = \frac{3}{2} K_{-}^{\frac{2}{k+1}}, \quad A = -\frac{1}{3} C \tag{93}$$

Therefore, the equivalent stress given by Eq. (21) may be expressed as follows:

$$\sigma_{\mathrm{e}} = \sqrt{\frac{2}{3} C} \sigma_{\mathrm{i}} + \alpha \sigma_1 \tag{94}$$

while the proposed Eqs. (31) and (44) together with relations (32), (33) and (46) can be rewritten as:

$$\dot{\varepsilon} = \frac{\sigma_{\mathrm{e}}^{k}}{\left(1 - \frac{\omega}{\omega^*}\right)^{m}} \left(\sqrt{\frac{3}{2} C} \frac{\mathbf{s}}{\sigma_{\mathrm{i}}} + \alpha B \mathbf{n} \otimes \mathbf{n} \right) \tag{95}$$

and

$$\dot{\boldsymbol{\omega}} = \frac{\sigma_{\mathrm{e}}^{k+1}}{\left(1 - \frac{\omega}{\omega^*}\right)^{m}} \mathbf{n} \otimes \mathbf{n} \tag{96}$$

It is clear that in order to describe creep and creep damage on the basis of Eqs. (95) and (96), it is necessary to use test data and damage data obtained from tension and compression tests. One also notes that Eq. (93) is a condition for the use of Eqs. (95) and (96) for practical purposes.

One now assumes the following data are obtained from basic experiments

$$K_{-} \neq K_{+}, \quad K_0 \neq \left[\left(\sqrt{3} - 1\right) K_{-}^{\frac{1}{k+1}} + K_{+}^{\frac{1}{k+1}} \right]^{k+1} \tag{97}$$

Obviously, this case is the most general case of material behavior during creep that is related to independent creep laws and damage laws for isotropic materials under tension, compression, and torsion. The creep behavior of these isotropic damaging materials can be described by the proposed Eqs. (31) and (44) together with relations (32), (33) and (46). The appropriate material parameters to be used in Eqs. (31) and (44) may be found from Eq. (81).

The above discussion for the particular cases of the evolution Eqs. (31) and (44) together with relations (32), (33) and (46) is summarized in Table 1.

7. Comparison of theoretical and experimental results

The model presented earlier in this paper is used in this section to predict the creep behavior under uniaxial nonproportional and multiaxial nonproportional loading, and under nonisothermal conditions. The corresponding theoretical results are compared with experimental results. The model parameters are first determined using the data from the basic experiments under uniaxial tension, uniaxial compression, and pure torsion for different values of constant stress at constant temperatures.

One considers the creep behavior of the aluminum alloy AK4-1T (Rubanov, 1987). The chemical composition of this material is 2.2Cu, 1.6Mg, 0.55Ni and 0.015Ti in weight percentages. Test specimens were taken from a plate with a thickness of 40 mm. One assumes that the plate material has three orthogonal directions of initial anisotropy and the stress axes coincide with the principal axes of anisotropy. Creep tests were conducted on specimens taken from the three principal directions of the plate as well as on specimens oriented at an angle of $\pi/4$ with respect to the longitudinal and transverse directions. Basic experiments in the normal direction of the plate were conducted on cylindrical specimens with a diameter of 12 mm and a gage length of 20 mm. All other test specimens were thin-walled tubes with an outside diameter of 20 mm, a wall thickness of 1 mm and a gage length of 35 mm. The initial isotropy of an aluminum alloy AK4-1T at temperatures of 423 K and 448 K was established according to Rubanov (1987).

Creep curves of the given material under uniaxial tension, uniaxial compression, and pure torsion for different values of constant stress at constant temperatures can be described on the basis of relations (63)–(68) with the values for the material constants taken as:

$$k = 13, \quad m = 3.5, \quad K_+ = 1.67 \times 10^{-35}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1}$$

$$K_- = 3.3 \times 10^{-36}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1}, \quad K_0 = 6.56 \times 10^{-32}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1} \tag{98}$$

at a temperature of $T = 448$ K, and

$$K = 25, \quad m = 3.5, \quad K_+ = 1.05 \times 10^{-65}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1}$$

$$K_- = 8.43 \times 10^{-67}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1}, \quad K_0 = 5.84 \times 10^{-59}\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1} \tag{99}$$

at a temperature of $T = 423$ K. It is seen from experimental data given in Eqs. (98) and (99) that material constant m does not depend on the temperature.

Table 2 provides the experimental values ω^* of the scalar damage parameter at the instant of failure in creep under uniaxial tension, uniaxial compression, and pure torsion, given by relations (48), (50) and (52), respectively. It is seen that the critical value ω^* varies from 10.0 to 38.2 $\mathrm{MJ/m^3}$. Thus, usually used condition (Zolochevsky, 1991) $\omega^* = \mathrm{const}$ is not true for the aluminum alloy under consideration. It is also found from the basic experiments (Table 2) that the amount of the damage at the instant of failure in creep can be defined as follows:

$$\omega^* = \sigma_\mathrm{i}^2(a - bI_1) \tag{100}$$

Table 1
Particular cases of evolution Eqs. (31)–(33), (44) and (46)

Equations	Experimental values for material constants		
	K_+	K_-	K_0
(31)–(33), (44), (46)	K_+	K_-	K_0
(85), (86)	K_+	K_+	$(\sqrt{3})^{k+1}K_+$
(90), (91)	K_+	K_+	K_0
(95), (96)	K_+	K_-	$[(\sqrt{3}-1)K_-^{\frac{1}{k+1}} + K_+^{\frac{1}{k+1}}]^{k+1}$

where

$$a = 4 \times 10^{-4}\ \mathrm{MPa}^{-1}, \quad b = 4 \times 10^{-7}\ \mathrm{MPa}^{-2} \tag{101}$$

Thus, the critical value ω^* of the scalar damage parameter depends on the type of loading, but does not depend on the temperature. By considering the unique scatter of test data for creep, particularly marked at the instant of failure, the agreement of the theoretical and experimental results in Table 2 may be considered satisfactory.

As can be seen from relations (98) and (99), at the same absolute value of the stress intensity given by Eq. (24) and at any given time, the creep strain intensity

$$\varepsilon_{\mathrm{i}} = \sqrt{\frac{2}{3}\zeta \cdot \cdot \zeta} \tag{102}$$

was largest under pure torsion and smallest under uniaxial compression. This also indicates the largest degree of the creep damage in pure torsion. Here

$$\zeta = \boldsymbol{\varepsilon} - \frac{1}{3}\varepsilon_0 \mathbf{I} \tag{103}$$

is the creep strain deviator, and

$$\varepsilon_0 = \boldsymbol{\varepsilon} \cdot \cdot \mathbf{I} \tag{104}$$

is the volumetric creep strain. Note also that for aluminum alloy AK4-1T at a temperature of 448 K, the difference in the creep strain rate between tension and compression even in the secondary stage is five times while at a temperature of 423 K the difference is 12.5 times. The creep behavior of this material in the tertiary creep phase is characterized by greater dependence on the loading type.

It is possible to describe the experimental data in Eqs. (98) and (99) by the following relations:

$$k = k_1 T + k_2, \quad m = 3.5, \quad K_+/Q = \exp(a_+ T + b_+)$$

$$K_-/Q = \exp(a_- T + b_-), \quad K_0/Q = \exp(a_0 T + b_0) \tag{105}$$

Table 2
Comparison of experimental and theoretical values of the scalar damage parameter at the instant of failure for an aluminum alloy AK4-1T in basic experiments at various states of stress and for various temperatures

T (K)	σ_{11} (MPa)	σ_{12} (MPa)	ω^* (MJ/m^3)	
			Experiment	Theory
423	250.0	0	17.6	18.8
423	260.0	0	18.5	20.0
423	270.0	0	26.0	21.3
448	200.0	0	10.0	12.8
448	226.0	0	14.8	15.8
423	−290.0	0	38.2	43.4
448	−230.0	0	25.2	26.0
423	0	155.0	31.0	28.8
448	0	117.5	18.0	16.6

where

$$Q = 1\ \mathrm{MPa}^{-k}\ \mathrm{h}^{-1}, \quad k_1 = -0.48\ \mathrm{K}^{-1}, \quad k_2 = 228.04, \quad a_+ = 2.78166\ \mathrm{K}^{-1}, \quad b_+ = -1326.26$$

$$a_- = 2.81769\ \mathrm{K}^{-1}, \quad b_- = -1344.02, \quad a_0 = 2.49144\ \mathrm{K}^{-1}, \quad b_0 = -1187.97 \tag{106}$$

One now considers the experimental data for this aluminum alloy under nonproportional loading conditions. Thin-walled tubular specimens with an outside diameter of 20 mm, a wall thickness of 1 mm, and a gage length of 35 mm are subjected to nonproportional loading by an axial (tensile or compressive) force together with (or without) torque.

First, one considers the creep behavior of the aluminum alloy under uniaxial nonproportional loading in isothermal process (Fig. 1(a) and (b)) at a temperature of 423 K and in nonisothermal processes (Figs. 2(a)–(c) and 3(a)–(c)). Circles in Figs. 1(b), 2(c) and 3(c) represent results of the tests for the aluminum alloy under consideration in the form of dependence $\omega - t$ for various values of the stress $\sigma_{11}(t)$ and temperature $T(t)$. Here solid line corresponds to computed data on the basis of Eqs. (31)–(33), (44), (46), (81) and (100) with the values for the materials parameters given in relations (101), (105) and (106). One assumes that microcracks form and propagate perpendicular to the axis of the specimen for the loading program presented in Fig. 1(a) and parallel to the axis of the specimen in the cases of nonproportional loading considered in Figs. 2(b) and 3(b).

Next, one considers the creep behavior of the aluminum alloy under multiaxial nonproportional loading in nonisothermal process considered in Fig. 4(a). Thin-walled tubes were subjected to torque according to the program presented in Fig. 4(b), while an axial loading considered in Fig. 4(c) was applied simultaneously. Results of this test are represented in Fig. 4(c) in the form of dependencies ε_{11} −

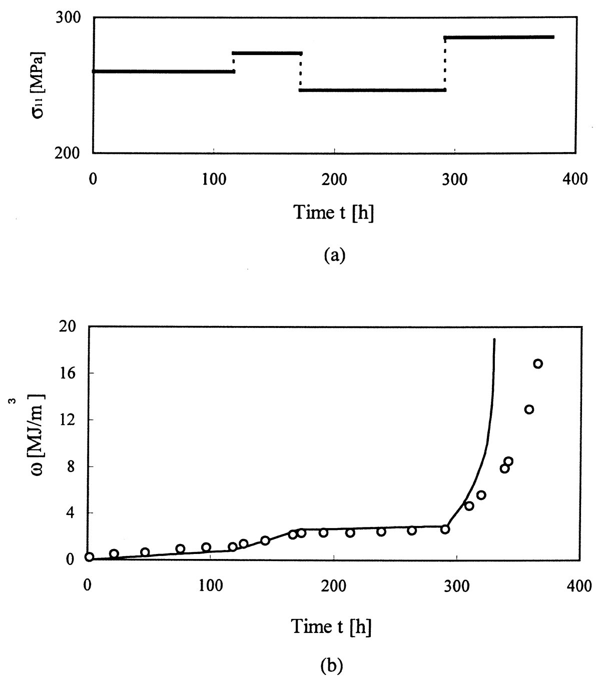

Fig. 1. Growth of damage variable $\omega_{11} = \omega$ for aluminum alloy AK4-1T at a temperature of 423 K (b) under nonproportional uniaxial tension for the loading program (a).

t (circles) and $2\varepsilon_{12} - t$ (triangles) for various values of stresses $\sigma_{11}(t)$ and $\sigma_{12}(t)$ and temperature $T(t)$. Solid and dashed lines correspond to results of calculations using Eqs. (31)–(33), (44), (46), (81) and (100) with the values for the materials parameters given in relations (101), (105) and (106); hence, axial strains and shear strains are indicated by the dashed line and the solid line, respectively. One assumes that microcracks in the case under consideration are orthogonal to the direction of the maximum principal stress. Thus, one can take

$$\mathbf{n} = [n_1, n_2, 0]^{\mathrm{T}} \tag{107}$$

where

$$n_1 = \sqrt{\frac{1}{2}\left(1 + \frac{\sigma_{11}}{\sqrt{\sigma_{11}^2 + 4\sigma_{12}^2}}\right)}, \quad n_2 = \sqrt{\frac{1}{2}\left(1 - \frac{\sigma_{11}}{\sqrt{\sigma_{11}^2 + 4\sigma_{12}^2}}\right)} \tag{108}$$

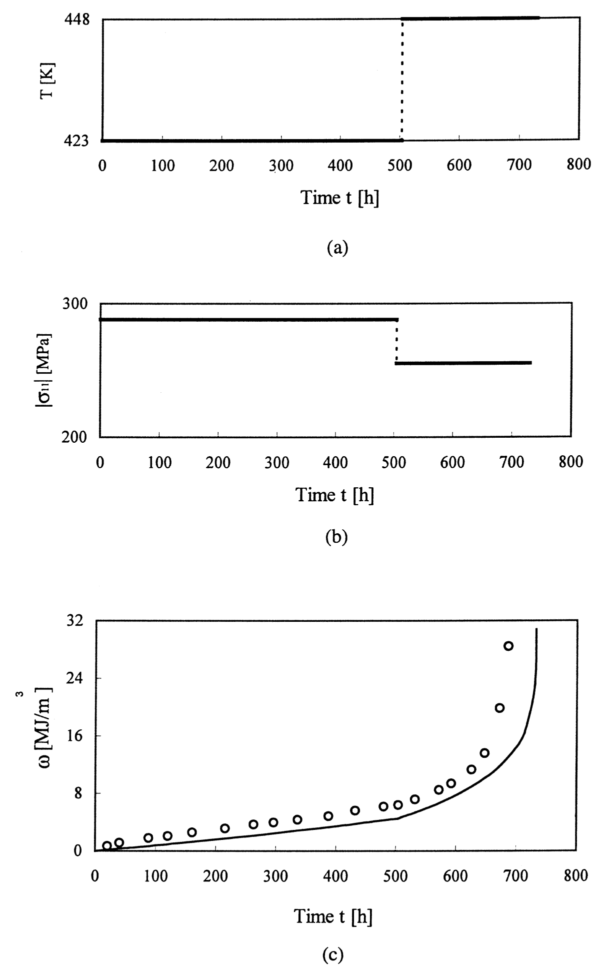

Fig. 2. Growth of damage variable $\omega_{22} = \omega$ for aluminum alloy AK4-1T (c) in nonisothermal process (a) under nonproportional uniaxial compression for the loading program (b).

By considering the unique scatter of test data for creep, particularly marked in the third stage, the agreement of the theoretical and experimental results under uniaxial nonproportional and multiaxial nonproportional loading, and for both isothermal and nonisothermal conditions may be considered satisfactory.

8. Conclusions

A creep damage analysis is proposed for the initially isotropic polycrystalline materials with different behavior in tension and compression, and without creep hardening. Creep damage in the materials under consideration is associated with the dislocation creep, and nucleation and growth of parallel flat microcracks. The classical thermodynamics of irreversible processes is used to describe different creep properties in tension and compression, damage induced anisotropy, and different damage development in tension and compression. The second-order damage tensor and scalar damage parameter is

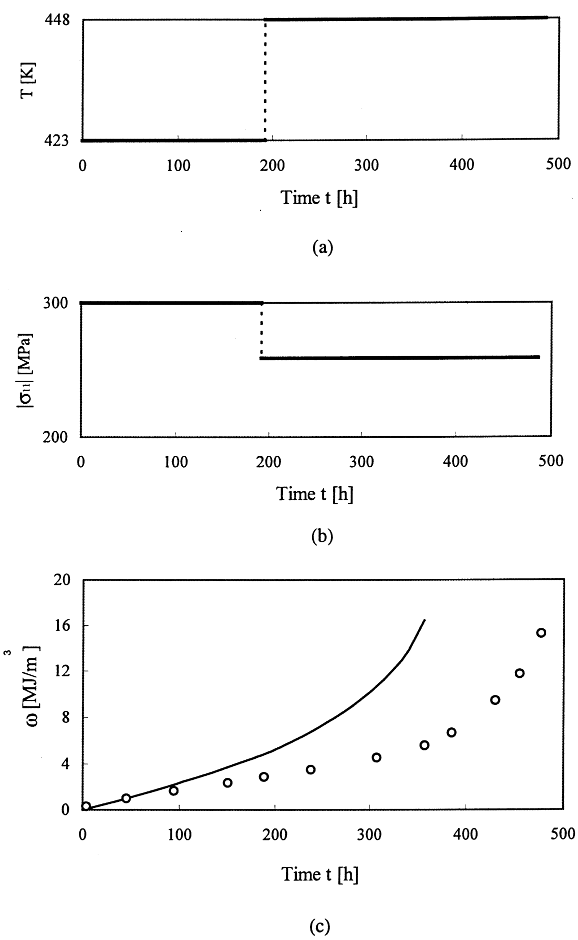

Fig. 3. Growth of damage variable $\omega_{22} = \omega$ for aluminum alloy AK4-1T (c) under nonproportional uniaxial compression conditions (b) and in nonisothermal process (a).

introduced to account for the creep damage. The equation for creep deformation and equation for damage evolution is derived.

The theoretical results have been compared with the experimental results under uniaxial nonproportional and multiaxial nonproportional loading. The model parameters are first determined from basic experiments under uniaxial tension, uniaxial compression, and pure torsion for different values of constant stress at constant temperatures. A good correlation is obtained between the results generated from the model and the experimental data of the creep on thin-walled tubular specimens subjected to an axial (tensile or compressive) force together with (or without) torque. The material

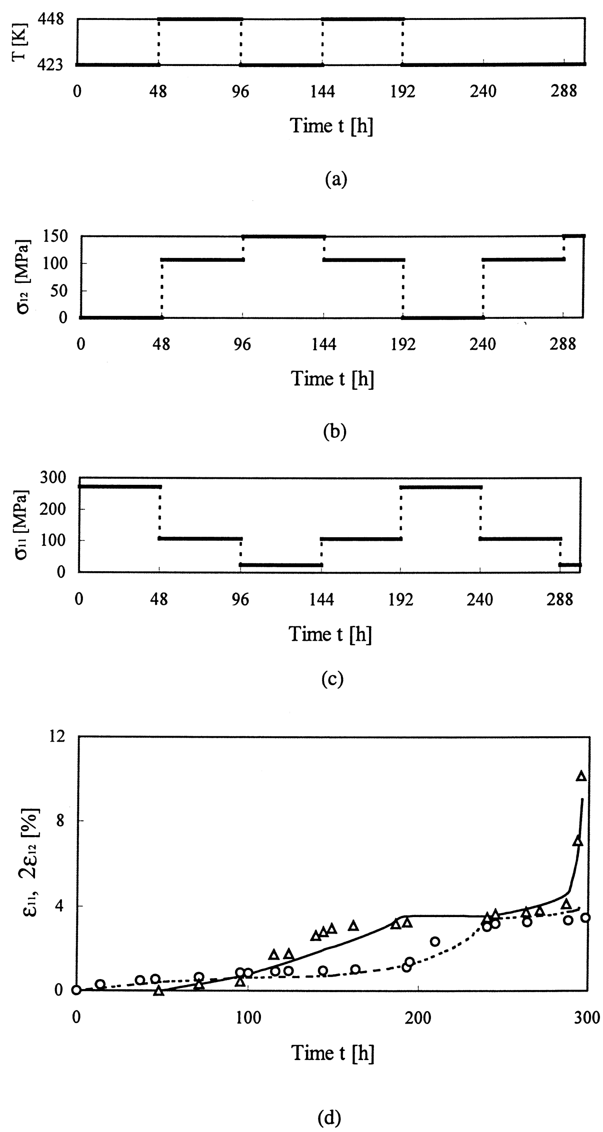

Fig. 4. Creep curves for aluminum alloy AK4-1T (d) obtained for nonproportional loading of the specimen that was subjected to torque according to the program (b) and to axial loading (c), and in nonisothermal process (a).

under consideration is the aluminum alloy under nonproportional loading in isothermal and nonisothermal conditions.

The description of a set of the additional physical phenomena occurring in polycrystalline materials, such as creep hardening, Bauschinger-type effect, cyclic creep, various memorization effects, etc. will be discussed in a forthcoming paper.

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